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Supergeometry of Three Dimensional Black Holes

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ABSTRACT: We show how the supersymmetric properties of three dimensional black holes can be obtained algebraically. The black hole solutions are constructed as quotients of the supergroup $OSp(1|2;R)$ by a discrete subgroup of its isometry supergroup. The generators of the action of the isometry supergroup which commute with these identifications are found. These yield the supersymmetries for the black hole as found in recent studies as well as the usual geometric isometries. It is also shown that in the limit of vanishing cosmological constant, the black hole vacuum becomes a null orbifold, a solution previously discussed in the context of string theory.

1. Introduction

Gravity in $2+1$ dimensions has no local dynamics. Classical solutions to the theory in the absence of matter are locally flat [1], or have constant curvature, if a cosmological constant is present [2]. Nevertheless, non-trivial global effects are possible and can yield interesting solutions such as black holes [3] [4]. A useful way of describing some vacuum solutions to $2+1$ gravity is in terms of a quotient construction. One begins with a simple symmetric space $\tilde{\mathcal{S}}$ and identifies points under the action of a discrete subgroup, I, of its isometry group, G, to obtain a spacetime, $\mathcal{S}$. The fixed points of the group action correspond to singularities of $\mathcal{S}$. The residual symmetry group of the spacetime is the subgroup $H \subset G$ that commutes with I.

In this paper, we study the supergeometry of the black hole solutions. In Section 2, we review the 2+1 dimensional black hole solutions focusing attention on their construction as quotients from the group manifold $SL(2,R)$. We also discuss how in the limit of vanishing cosmological constant, the $M = J = 0$ black hole vacuum becomes the null orbifold of string theory. In Section 3, the solutions are imbedded in the supergroup $OSp(1|\,2;R)$. The generators of the action of the isometry supergroup which commute with the black hole identifications are found. The even generators yield Killing vectors. The odd generators can be put into correspondence with two component spinors. We obtain the same number of Killing vectors and spinors as found in studies of their supersymmetric properties [5][6] in the context of $2+1$ dimensional anti-deSitter supergravity [7].

2. 2+1 Dimensional Black Hole Solutions

$2+1$ dimensional black holes [3] are solutions to Einstein's equations with a negative cosmological constant, Λ,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \quad \Lambda < 0. \tag{2.1}$$

The metric for the black hole solutions is given by

$$ds^2 = -(\frac{r^2}{l^2} - M)dt^2 - J dt d\phi + (\frac{r^2}{l^2} - M + \frac{J^2}{4r^2})^{-1} dr^2 + r^2 d\phi^2, \quad 0 \le \phi < 2\pi \tag{2.2}$$

where $l \equiv (-\Lambda)^{-1/2}$.* M and J are the mass and angular momentum. (2.2) describes a black hole solution with outer and inner horizons at $r = r_+$ and $r = r_-$ respectively where

$$r_\pm = l(\frac{M}{2})^{1/2}\Big(1 \pm \Big(1 - (\frac{J}{Ml})^2\Big)^{1/2}\Big)^{1/2}. \tag{2.3}$$

* We have set $G = 1/8$.

The region $r_+ < r < M^{1/2}l$ defines an ergosphere, in which the asymptotic timelike Killing field $\frac{\partial}{\partial t}$ becomes spacelike. $M = J = 0$ is the black hole vacuum. The solutions with $-1 < M < 0,\ J = 0$ describe point particle sources with naked conical singularities at $r = 0$ [2]. The solution with $M = -1,\ J = 0$ is anti-deSitter space.

We now review the construction of the black hole solutions as quotients of three dimensional anti-deSitter space [3]. It will be more useful for the later discussion to view three dimensional anti-deSitter space as the group manifold $SL(2,R)$ and the group of identifications as a discrete subgroup of $SL(2,R)_L \otimes SL(2,R)_R$, the isometry group of $SL(2,R)$. Every solution to (2.1) in 2+1 dimensions corresponds to three dimensional anti-deSitter space *locally.* However, since one is still free to make discrete identifications, the solution can differ *globally.* Three dimensional anti-deSitter space is most easily described in terms of the three dimensional hypersurface

$$-T^2 + X^2 - W^2 + Y^2 = -l^2 \tag{2.4}$$

imbedded in the four dimensional flat space with metric

$$ds^2 = -dT^2 + dX^2 - dW^2 + dY^2. \tag{2.5}$$

The topology of (2.4) is $R^2 \times S^1$ with S^1 corresponding to the timelike circles $T^2 + W^2 = const$. Anti-deSitter space is the covering space obtained by unwinding the circle.

The isometry group of three dimensional anti-deSitter space is the subgroup of the isometry group of the flat space (2.5) which leaves (2.4) invariant. This is $SO(2,2)$ with rotations in the $T - W$ plane which differ by $2\pi n$ *not* identified. The hypersurface (2.4) describing three dimensional anti-deSitter space is the group manifold of $SL(2,R)$ as can be seen from the imbedding

$$g = \frac{1}{l}\begin{pmatrix} T+X & Y-W \\ Y+W & T-X \end{pmatrix}, \quad \det g = (T^2 - X^2 + W^2 - Y^2)/l^2 = 1. \tag{2.6}$$

The metric (2.5) is the bi-invariant metric

$$ds^2 = \frac{l^2}{2}\mathrm{Tr}(g^{-1}dg)^2 \ . \tag{2.7}$$

In this representation, the $SO(2,2)$ isometries are induced by its two fold cover $SL(2,R)_L \otimes SL(2,R)_R$:

$$g \to AgB,\ \ A, B \in SL(2,R). \tag{2.8}$$

It is a two-fold cover because (A,B) and $(-A,-B)$ induce the same element of $SO(2,2)$. We can choose

$$L_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad L_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad L_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{2.9}$$

as generators of $SL(2,R)$. The black hole solutions are obtained by identifying points in anti-deSitter space under the action of a discrete subgroup of $SL(2,R)_L \otimes SL(2,R)_R$.

2.1. $M > 0$: Black Hole Solutions

The $M > 0$ black hole solutions are obtained by the identification [3] [4]

$$g \sim A^n g B^n, \quad n \text{ integer} \tag{2.10}$$

where

$$\begin{aligned} A &= \exp\left(\pi \frac{(r_+ + r_-)}{l} L_3\right) = \begin{pmatrix} e^{\pi(r_+ + r_-)/l} & 0 \\ 0 & e^{-\pi(r_+ + r_-)/l} \end{pmatrix}, \\ B &= \exp\left(\pi \frac{(r_+ - r_-)}{l} L_3\right) = \begin{pmatrix} e^{\pi(r_+ - r_-)/l} & 0 \\ 0 & e^{-\pi(r_+ - r_-)/l} \end{pmatrix}, \end{aligned} \tag{2.11}$$

with L_3 given in (2.9) and $r_\pm$ given in (2.3). This identification is generated by

$$L = \frac{(r_+ + r_-)}{l} L_3^L + \frac{(r_+ - r_-)}{l} L_3^R \in sl(2,R)_L \oplus sl(2,R)_R, \qquad M > |J|/l. \tag{2.12}$$

For $J \neq 0$, g has no fixed points under the action of (2.11) consistent with the fact that the rotating black hole solution is non-singular. However, for the non-rotating ($J = 0$) black hole, there are fixed points under the identification (2.10) which correspond to the singularity $r = 0$.

2.2. Black Hole Vacuum

The $M = J = 0$ black vacuum is given by

$$ds^2 = -\frac{r^2}{l^2} dt^2 + \frac{l^2}{r^2} dr^2 + r^2 d\phi^2, \quad 0 < \phi < 2\pi. \tag{2.13}$$

We first obtain its $\Lambda \to 0$ ($l \to \infty$) limit which should describe a locally flat metric. Define the new coordinate

$$v = 2t + 2l^2/r, \tag{2.14}$$

which parameterizes outgoing null curves. The metric (2.13) then becomes

$$ds^2 = -\frac{r^2}{4l^2} dv^2 - dv dr + r^2 d\phi^2, \quad 0 < \phi < 2\pi. \tag{2.15}$$

Now, as $l \to \infty$, (2.15) has the smooth limit

$$ds^2 = -dv dr + r^2 d\phi^2, \quad 0 < \phi < 2\pi. \tag{2.16}$$

(2.16) is the metric for a *null orbifold* and has been considered previously in the context of string theory [8]. It has zero curvature and can be obtained by identifying three-dimensional Minkowski space under the action of a null boost.

Like the null orbifold, the black hole vacuum can be obtained by identifying points under the action of a null boost, but now in three dimensional anti-deSitter space rather than flat space. Consider coordinates in three dimensional anti-deSitter space defined by the following imbedding

$$\begin{aligned} U &\equiv T - X = r \\ V &\equiv T + X = v - \frac{rv^2}{4l^2} + r\phi^2 \\ W &= \frac{vr}{2l} - l \\ Y &= r\phi. \end{aligned} \tag{2.17}$$

Translations ($\phi \to \phi + E$) correspond to null boosts in (U, V, Y)

$$N_E : \qquad \begin{aligned} U &\to U' = U \\ V &\to V' = V + 2EY + E^2 U \\ Y &\to Y' = Y + EU \\ W &\to W' = W. \end{aligned} \tag{2.18}$$

N_E can be obtained by a contraction, *i.e.* by conjugating a Euclidean rotation of angle θ by a boost of velocity β in the simultaneous limit that $\beta \to 1$ and $\theta \to 0$ with $E = \theta/\sqrt{1-\beta^2}$ held. r in (2.17) labels the $U = const.$ null surfaces which N_E leaves invariant. Identifying points under the action of

$$I = \{N_{2\pi n},\ n \text{ integer}\} \tag{2.19}$$

corresponds to making ϕ periodic in 2π. Substituting (2.17) into (2.5), we obtain the black hole vacuum (2.15). Translations in v also preserve the metric (2.15) and correspond to null boosts in the (U, V, W) space with Y fixed. The set of fixed points of (2.18) are

$$\mathcal{L} = \{U = Y = 0, W = -l\} \tag{2.20}$$

and from (2.17) is seen to correspond to the null singularity $r = 0$.

From (2.18) (2.6), the black hole vacuum is thus obtained by the identification

$$g \sim A^n g B^n, \quad n \text{ integer}. \tag{2.21}$$

where

$$A = \exp 2\pi L_+ = \begin{pmatrix} 1 & 2\pi \\ 0 & 1 \end{pmatrix}, \quad B = \exp 2\pi L_- = \begin{pmatrix} 1 & 0 \\ 2\pi & 1 \end{pmatrix} \tag{2.22}$$

generated by

$$L_+^L + L_-^R \in sl(2,R)_L \oplus sl(2,R)_R, \qquad \text{Black Hole Vacuum.} \tag{2.23}$$

2.3. Extremal $M = |J|/l$ Solution

In this section, we obtain the extremal $M = |J|/l$ solution as a quotient by a discrete subgroup of $SL(2,R)_L \otimes SL(2,R)_R$. We first review how the solutions are constructed by identifying points in anti-deSitter space [3]. Setting $M = J/l$ in (2.2) yields the extremal solution

$$ds^2 = -(\frac{r^2}{l^2} - M)dt^2 - Mldtd\phi + \frac{dr^2}{(\frac{r}{l} - \frac{Ml}{2r})^2} + r^2 d\phi^2, \quad 0 < \phi < 2\pi \tag{2.24}$$

The case $M = -J/l$ can be obtained by letting $t \to -t$.

It is useful to consider Poincare coordinates [9] $(\lambda_+, \lambda_-, z)$ defined by the imbedding

$$\begin{aligned} T + X &= l/z \\ T - X &= l(z + (\lambda_+\lambda_-)/z) \\ W &= -\frac{\lambda_+ - \lambda_-}{2z} l \\ Y &= \frac{\lambda_+ + \lambda_-}{2z} l. \end{aligned} \tag{2.25}$$

Using (2.5), the metric for anti-deSitter space in Poincare coordinates takes the form

$$ds^2 = \frac{l^2}{z^2}(d\lambda_+ d\lambda_- + dz^2). \tag{2.26}$$

Consider the one-parameter subgroup of $SO(2,2)$ transformations with parameter χ

$$\begin{aligned} \lambda_+ &\to \lambda_+ + \chi \\ \lambda_- &\to e^{(2M)^{1/2}\chi}\lambda_- + (2M)^{-1/2}(e^{(2M)^{1/2}\chi} - 1) \\ z &\to e^{(M/2)^{1/2}\chi} z \end{aligned} \tag{2.27}$$

leaving (2.26) invariant. It was shown in [3] that the extremal black hole (2.24) is obtained by identifying under (2.27) with $\chi = 2\pi$. From (2.25) and (2.6), the extremal black hole is obtained by the $SL(2,R)_L \otimes SL(2,R)_R$, identification

$$g \sim A^n g B^n, \quad n \text{ integer} \tag{2.28}$$

where

$$A = \begin{pmatrix} e^{-(2M)^{1/2}\pi} & 0 \\ (2/M)^{1/2}\sinh\left((2M)^{1/2}\pi\right) & e^{(2M)^{1/2}\pi} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2\pi \\ 0 & 1 \end{pmatrix} \tag{2.29}$$

generated by

$$L_-^L - (\frac{M}{2})^{1/2} L_3^L + L_+^R \in sl(2,R)_L \oplus sl(2,R)_R, \qquad \text{Extremal Black Hole.} \tag{2.30}$$

2.4. $M < 0,\ J = 0$ Solutions with Naked Singularities

For the $M < 0, J = 0$ solution, it is convenient to use static coordinates defined by the imbedding

$$\begin{aligned} T &= \sqrt{\tilde{r}^2 + l^2}\cos\tilde{t}/l, & W &= \sqrt{\tilde{r}^2 + l^2}\sin\tilde{t}/l, \\ X &= \tilde{r}\cos\tilde{\phi}, & Y &= \tilde{r}\sin\tilde{\phi}, \end{aligned} \tag{2.31}$$

in terms of which the metric (2.5) for three dimensional anti-deSitter space takes the form

$$ds^2 = -(\frac{\tilde{r}^2}{l^2} + 1)d\tilde{t}^2 + (\frac{\tilde{r}^2}{l^2} + 1)^{-1}d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2. \tag{2.32}$$

$\tilde{t}$ and $\tilde{\phi}$ now parameterize rotations in the $T - W$ and $X - Y$ planes. The solution is now obtained by identifying $\tilde{\phi}$ periodically with period $2\pi\sqrt{|M|}$. Rescaling the coordinates

$$\tilde{r} = r/\sqrt{|M|}, \quad \tilde{t} = \sqrt{|M|}\,t, \quad \tilde{\phi} = \sqrt{|M|}\,\phi, \tag{2.33}$$

one obtains (2.2) where ϕ has canonical period 2π.

From (2.6), a rotation of angle θ in the $X - Y$ plane takes the form (2.8)

$$g \to \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix} g \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{pmatrix}. \tag{2.34}$$

Hence, the $M < 0, J = 0$ solution is obtained by the identification

$$g \sim A^{-n} g A^n \tag{2.35}$$

where

$$A = \exp\left(\pi\sqrt{|M|}(L_+ - L_-)\right) = \begin{pmatrix} \cos\pi\sqrt{|M|} & \sin\pi\sqrt{|M|} \\ -\sin\pi\sqrt{|M|} & \cos\pi\sqrt{|M|} \end{pmatrix}. \tag{2.36}$$

generated by

$$L_+^L - L_-^L - L_+^R + L_-^R \in sl(2,R)_L \oplus sl(2,R)_R, \qquad \text{M} < 0,\ \text{J} = 0. \tag{2.37}$$

The fixed points of the group action are $\{X = Y = 0\}$, and from (2.31) is seen to correspond to the singularity $r = 0$. These solutions are the anti-deSitter analog of the conical solution [1] and were first constructed in [2].

3. Supergeometry

In this section, we study the supergeometry of the black hole solutions. After imbedding the black hole spacetime in the supergroup $OSp(1|\,2;R)$, one finds the generators of the isometry group of the supergroup which commute with the black hole identifications. The even generators yield the usual Killing vectors. However, in addition, there are odd generators of the isometry group of $OSp(1|\,2;R)$ which are consistent with the black hole identifications. These can be put into correspondence with two-component spinors. We find the same number of these Killing spinors as were found in studies of their supersymmetric properties [5][6]. In [5], it was pointed out that the Killing spinors in the black hole are those in anti-deSitter space which respect the identifications. Let us now review the construction of the supergroup $OSp(1|\,2;R)$.

3.1. $OSp(1|\,2;R)$

Consider a Grassmann algebra, $\mathcal{A}$, generated by one Grassmann element, ϵ

$$\mathcal{A} = \{z = a + b\epsilon, \quad a, b \in R, \quad \epsilon^2 = 0\}. \tag{3.1}$$

a and $b\epsilon$ are the even and odd parts of z. $OSp(1|\,2;R)$ is the set of linear transformations of (θ^1, θ^2, x) of the form

$$OSp(1|\,2;R) = \left\{ M = \begin{pmatrix} a & b & \alpha \\ c & d & \beta \\ \gamma & \delta & 1 \end{pmatrix}, \quad a, \ldots \text{ even}, \quad \alpha, \ldots \text{ odd} \right\} \tag{3.2}$$

which preserve

$$dl^2 = \epsilon_{ab}\theta^a\theta^b + x^2, \quad \epsilon_{12} = -\epsilon_{21} = 1 \tag{3.3}$$

and where θ^1, θ^2 are Grassmannian satisfying

$$\theta^1\theta^1 = \theta^2\theta^2 = 0, \; \{\theta^1, \theta^2\} = 0, \; \{\epsilon, \theta^a\} = 0. \tag{3.4}$$

The condition that M preserves the line element (3.3) implies the relations

$$\begin{aligned} ad - bc &= 1 \\ c\alpha - a\beta &= -\gamma \\ d\alpha - b\beta &= -\delta. \end{aligned} \tag{3.5}$$

Since these are three relations for 8 parameters, $OSp(1|\,2;R)$ is five dimensional. $OSp(1|\,2;R)$ contains $SL(2,R)$ as a subgroup

$$SL(2,R) \simeq Sp(2,R) \simeq \left\{ g = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad ad - bc = 1 \right\} \subset OSp(1|\,2;R). \tag{3.6}$$

Consider the following basis for the Lie algebra $osp(1|2;R)$. The even generators are those in the $sl(2,R)$ subalgebra and are given by (2.9)

$$L_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad L_+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad L_- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.7}$$

and the odd generators are

$$Q_+ = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \quad Q_- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{3.8}$$

They satisfy the algebra

$$\begin{aligned} &[L_3, L_+] = L_+, \quad [L_3, L_-] = -L_-, \quad [L_+, L_-] = L_3 \\ &[L_3, Q_+] = Q_+, \quad [L_+, Q_+] = 0, \quad [L_-, Q_+] = Q_- \\ &[L_3, Q_-] = -Q_-, \quad [L_+, Q_-] = Q_+, \quad [L_-, Q_-] = 0 \\ &\{Q_+, Q_+\} = -2L_+, \quad \{Q_-, Q_-\} = 2L_-, \quad \{Q_+, Q_-\} = L_3. \end{aligned} \tag{3.9}$$

As we now show, the adjoint action of the $SL(2,R)$ subgroup induces an $SO(2,1)$ transformation on the $sl(2,R)$ subalgebra and an $SL(2,R)$ transformation on the odd generators $Q_\pm$. Consider the adjoint action by an element

$$h = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix} \in SL(2,R) \tag{3.10}$$

on the Lie algebra $osp(1,2|R)$. On the $sl(2,R)$ subalgebra, the adjoint action

$$ad_h : L \to h^{-1} L h \tag{3.11}$$

induces the transformation on the basis (3.7)

$$\begin{aligned} L_3 &\to (ad + bc)L_3 + 2bdL_+ - 2acL_- \\ L_+ &\to cdL_3 + d^2 L_+ - c^2 L_- \\ L_- &\to -abL_3 - b^2 L_+ + a^2 L_-. \end{aligned} \tag{3.12}$$

This is an $SO(2,1)$ transformation preserving inner product

$$< A, B >= \frac{l^2}{2} Tr(AB) \tag{3.13}$$

with h and $-h$ inducing the same element of $SO(2,1)$. Under the adjoint action (3.11), the odd generators (3.8) transform as

$$Q_+ \to ad_h Q_+ = h^{-1} Q_+ h = dQ_+ - cQ_-, \quad Q_- \to ad_h Q_- = h^{-1} Q_- h = -bQ_+ + aQ_- \tag{3.14}$$

corresponding to the $SL(2,R)$transformation

$$\begin{pmatrix} Q_+ \\ Q_- \end{pmatrix} \to (h^{-1})^t \begin{pmatrix} Q_+ \\ Q_- \end{pmatrix}. \tag{3.15}$$

A vector on the $SL(2,R)$ submanifold of $OSp(1|\,2;R)$ at the point g (3.6) can be decomposed into a vector w tangent to $SL(2,R)$ and a transverse odd vector field ψ

$$v = w + \psi, \quad \psi = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & \beta \\ \gamma & \delta & 0 \end{pmatrix} \tag{3.16}$$

with $\alpha, \ldots$ odd and satisfying (3.5). We associate with each odd vector field ψ (3.16), the spinor field

$$\psi = \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}, \quad \alpha = \bar{a}\epsilon, \ \beta = \bar{b}\epsilon. \tag{3.17}$$

A right invariant basis of vector fields for $OSp(1|\,2;R)$ on the $SL(2,R)$ submanifold can be obtained by left multiplication of (3.6) by the generators (3.7) and (3.8). The three vector fields obtained from (3.7) are a right invariant basis of vector fields tangent to $SL(2,R)$ while the two odd vectors obtained from (3.8) yields the right invariant basis of odd vector fields given by

$$\psi_+ = Q_+ g = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -c & -d & 0 \end{pmatrix} \quad \psi_- = Q_- g = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ a & b & 0 \end{pmatrix} \tag{3.18}$$

with corresponding spinors

$$\psi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \psi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3.19}$$

using (3.16) and (3.17).

3.2. Supersymmetries

Since $SL(2,R)$ is a subgroup of $OSp(1|\,2;R)$, the black hole solutions which are constructed as quotients of $SL(2,R)$ can also be viewed as quotients of $OSp(1|\,2;R)$. Since $OSp(1|\,2;R)$ is a group, its symmetry group with respect to a bi-invariant metric is $OSp(1|\,2;R)_L \otimes OSp(1|\,2;R)_R$. The symmetry group of the quotient $OSp(1|\,2;R)/I$ is H where H is the subgroup of $OSp(1|\,2;R)_L \otimes OSp(1|\,2;R)_R$ commuting with I

$$[H,I]=0, \quad H \subset OSp(1|\,2;R)_L \otimes OSp(1|\,2;R)_R. \tag{3.20}$$

The even generators of the Lie algebra of H, $\mathcal{H}$, are the usual Killing symmetries while the odd generators are the supersymmetries or Killing spinors. Given the odd generators, the corresponding Killing spinor fields can be obtained by left or right multiplication by g in (3.6). For the case of anti-deSitter space with no quotient taken ($I=1$), the full symmetry group is $H \in OSp(1|\,2;R)_L \otimes OSp(1|\,2;R)_R$ yielding $2\times 3=6$ Killing vectors and $2\times 2=4$ supersymmetries. Now we consider the black hole solutions.

From (3.9), we find that for the non-extremal black hole, there are two generators commuting with I (2.12)

$$L_3^L, L_3^R \in \mathcal{H} \quad (\mathrm{Non-Extremal\ Black\ Hole}) \tag{3.21}$$

implying there are

$$2 \text{ Killing vectors and } 0 \text{ Killing spinors} \quad (\mathrm{Non-Extremal\ Black\ Hole}). \tag{3.22}$$

There are no Killing spinors because no non-trivial linear combination of $Q_\pm$ commutes with L_3.

For the black hole vacuum, there are four generators commuting with I (2.23)

$$L_+^L, L_-^R, Q_+^L, Q_-^R, \quad (\mathrm{Vacuum}) \tag{3.23}$$

implying

$$2 \text{ Killing vectors and } 2 \text{ Killing spinors} \quad (\mathrm{Vacuum}). \tag{3.24}$$

For the extremal black hole solutions, using (3.9) we find that there are three generators commuting with (2.30)

$$L_-^L - (M/2)^{1/2} L_3^L, L_+^R, Q_+^R, \quad (\mathrm{Extremal\ Black\ Hole}) \tag{3.25}$$

implying

$$2 \text{ Killing vectors and 1 Killing spinor}, \quad (\text{Extremal Black Hole}). \tag{3.26}$$

From (3.9), we find that for the $M < 0$ solutions, there are two generators commuting with I (2.37)

$$L_+^L - L_-^L, L_+^R - L_-^R, \quad (-1 < M < 0,\ J = 0) \tag{3.27}$$

implying

$$2 \text{ Killing vectors and 0 Killing spinors}, \quad (-1 < M < 0,\ J = 0). \tag{3.28}$$

There are no Killing spinors because no non-trivial linear combination of $Q_\pm$ commutes with $L_+ - L_-$. For all the black hole solutions, the two Killing vectors correspond to linear combinations of $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \phi}$.

We can also recover the Killing vectors and spinors for the self-dual backgrounds considered in [10]. The group of identifications for a causally well-behaved self-dual solution is a subgroup of one of the $SL(2,R)$ factors, say $SL(2,R)_L$, generated by a spacelike generator. Since the left and right factors commute, there are two Killing spinors and three Killing vectors coming from $OSp(1,2|R)_R$. From $SL(2,R)_L$, there are zero Killing spinors and one Killing vector. Hence, for the self-dual solution there are in total four Killing vectors and two Killing spinors.

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